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Existence of Solutions for Schrödinger Equations with a Point Interaction

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Resumen

We study the existence, uniqueness and regularity of solutions for the initial value problem for the time dependent Schrödinger equation with a point interaction,

$$\begin{cases} \partial_t u = i\left(\Delta_Z + V(x,t)u\right), \quad x \in \mathbb{R}, \quad t \in \mathbb{R} \\ u(x,s) = u_0, \end{cases} \tag{1}$$

Palabras & frases claves: Schrodinger equations, point interaction, delta-interaction.

1. Introducción

We study the existence, uniqueness and regularity of solutions for the initial value problem for the time dependent Schrödinger equation with a point interaction,

$$\begin{cases} \partial_t u = i\left(\Delta_Z + V(x,t)u\right), \quad x \in \mathbb{R}, \quad t \in \mathbb{R} \\ u(x,s) = u_0, \end{cases} \tag{2}$$

where $V = V(x,t)$ is real value function and $-\Delta_Z$ is the operator formally written

$$-\Delta_Z = -\frac{d^2}{dx^2} + Z\delta_0,$$

with δ_0 being the Dirac's delta centered at zero and Z is a real number. We give sufficient conditions on $V(x,t)$ such that equation (2) uniquely generates

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a strongly continuous unitary propagator $U(t,s)$ on the space $D(-\Delta_Z)$, which is the domain for operator $-\Delta_Z$ and such that $U(t,s)u_0 \in C(I, D(-\Delta_Z)) \cap C^1(I, L^2(\mathbb{R}))$ for every $u_0 \in D(-\Delta_Z)$.

The ideas used by Yajima in [8] were our motivation. The first, we show that the operator

$$Q_Z u(t) = \int_0^t \exp(i\Delta_Z(t-s))V(x,s)u(s)\ ds \tag{3}$$

is a contraction on the Banach space $\mathfrak{X}(a,l) = C([-a,a]; L^2(\mathbb{R})) \cap L^{l,\theta}([-a,a])$, for certain parameters a, l e $\theta \in \mathbb{R}$. The condition on potential V is given by $V \in \mathfrak{M} = L^{p,\alpha}([-a,a]) + L^{\infty,\beta}([-a,a])$. Then, we obtain existence and uniqueness of solution of the problem (2) in $L^2(\mathbb{R})$. If the potential V and the initial condition u_0 are more regular, then the solution u corresponding in $L^2(\mathbb{R})$, has the same regularity of inicial data u_0. The second, we show that L^2-norm of solution u is a conserved quantity.

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