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On the dynamics of periodic orbits of finite order for a particular homeomorphism of the annulus

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Resumen

Let $f : \mathbb{A} \longrightarrow \mathbb{A}$ be the homeomorphism on the annulus $\mathbb{A} = \mathbb{S}^1 \times [0, 1]$ defined in [1]. We know that any periodic orbit $\mathcal{O}$ of period q and rotation number $0 < p/q \leqslant 1$ can be arranged as a positive braid, and there are only two of them order preserving for each rotation number. In [2] Boyland gave the definition of a (p, q)-topologically monotone periodic orbit for annulus homeomorphims. We show that a periodic orbit is order preserving if and only if, this is a finite order periodic orbit. To this, we consider a simple closed curve $\gamma \subset \mathbb{A}$ contain $\mathcal{O}$ that generates the homology of $\mathbb{A}$, which is invariant by the action of the homeomorphism f.

Palabras & frases claves: Rotation number, periodic orbit, topologically monotone, finite order

1. Introducción

The main results in this talk will be:

Theorem 1. *Let $f : \mathbb{A} \longrightarrow \mathbb{A}$ be a homeomorphism on the annulus $\mathbb{A} = \mathbb{S} \times [0, 1]$, and $\mathcal{O}$ a f-periodic orbit. Suppose that γ is a nullhomotopic simple closed curve on $\mathbb{A}$, that generates the homology of $\mathbb{A}$ and*

1. $\mathcal{O} \subset \gamma$.

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2. $f(\gamma)$ is a homotopic to γ on $\mathbb{A} \setminus \mathcal{O}$.

Then, $\mathcal{O}$ is a finite order periodic orbit of f, that is f is isotopic to a rotation relative to $\mathcal{O}_f(x)$ (coordinate change).

Theorem 2. *Let $f : \mathbb{A} \longrightarrow \mathbb{A}$ be the homeomorphism on the annulus $\mathbb{A} = \mathbb{S}^1 \times [0, 1]$ isotopic to the identity define in [1]. Let F be a lift to the universal cover $\overline{\mathbb{A}} = \mathbb{R}\times[0, 1]$. If $\mathcal{O}$ is a f-periodic orbit of period q and rotation number $0 < p/q < 1$, then $\mathcal{O}$ is a finite order periodic orbit if and only if $\overline{\mathcal{O}} = \pi_x \circ \{F^n(\bar{x})\}_{n=0}^{\infty}$ is order-preserving.*

Referencias

[1] P. J. Holmes. Knotted periodic orbits in suspension annulus maps. Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, 411 (1987) 351-378.

[2] P. Boyland. Rotation sets and monotone periodic orbits for annulus homeomorphisms. Springer Commentarii Mathematici Helvetici , 67 (1992) 203-213.